FULLY WORKED SOLUTIONS

Context 4: The physics of fun

Chapter 11: Hang on!

Chapter questions

1. (a) $W = Fs = 300 \times 5 = 1500$ J

   (b) $W = Fs = mgs = 20 \times 9.8 \times 2 = 392$ J

   (c) 0

2. (a) $W = Fs = mgs = 30 \times 9.8 \times 3.2 = 940$ J

   (b) –940 J

3. $W = Fs \cos \theta = 10 \times 12 \times \cos 40° = 92$ J

4. (a) $0 = mg\Delta h + ½m (v^2 - u^2) + F_f d$

   $0 = (0.1 \times 9.8 \times 2.8) + ½ \times 0.1 \times (0 - 8^2) + 2.8F_f$

   $0 = 2.744 - 3.2 + 2.8F_f$

   $0.456 = 2.8F_f$

   $F_f = 0.16$ N

   (b) $0 = (0.1 \times 9.8 \times -2.8) + ½ \times 0.1 \times (v^2 - 0^2) + 0.163 \times 2.8$

   $0 = -2.744 + 0.05v^2 + 0.456$

   $2.29 = 0.05v^2$

   $v = 6.8 \text{ m s}^{-1}$

5. $d = 2.5/\sin 37° = 4.15$ m

   (a) $0 = mg\Delta h + ½m(v^2 - u^2) + F_f d$

   $0 = (40 \times 9.8 \times (-2.5)) + ½ \times 40 \times v^2 + 12 \times 4.15$

   $0 = -980 + 20v^2 + 49.8$

   $930.2 = 20v^2$

   $v = 6.8 \text{ m s}^{-1}$

   (b) $0 = (40 \times 9.8 \times (-1.25)) + ½ \times 40 \times v^2 + 12 \times 2.08$

   $0 = -490 + 20v^2 + 25$

   $465 = 20 v^2$

   $v = 4.8 \text{ m s}^{-1}$

6. (a) $E_k = 0$

   (b) $0 = mg\Delta h + ½m(v^2 - u^2)$

   $0 = (60 \times 9.8 \times (-2.5)) + ½ \times 60 \times v^2$

   $1470 = 30 v^2$

$v = 7 \text{ m s}^{-1}$

(c) $E_{gp} = mgh = 60 \times 9.8 \times 1.5 = 882$ J

(d) $0 = (60 \times 9.8 \times -1.5) + ½ \times 60 v^2$

$882 = 30 v^2$

$v = 5.4 \text{ m s}^{-1}$

7. (a) 0 effectively

(b) $0 = mg\Delta h + ½m(v^2 - u^2)$

$0 = 3000 \times 9.8 \times -35 + ½ \times 3000v^2$

$1.029 \times 10^6 = 1500 v^2$

$v = 26.2 \text{ m s}^{-1}$

(c) $0 = (3000 \times 9.8 \times 18) + ½ \times 3000 \times (v^2 - 26.2^2)$

$0 = 529\,200 + 1500v^2 - 1.03 \times 10^6$

$5 \times 10^5 = 1500v^2$

$v = 18.3 \text{ m s}^{-1}$

8. (a) $W = \Delta E_{gp} = mg\Delta h = 3200 \times 9.8 \times 45 = 1.4 \times 10^6$ J

(b) $t = W/P = 1.4 \times 10^6/1.24 \times 10^4 = 114$ s

9. (a) $W = \Delta E_{gp} = mg\Delta h = 2200 \times 9.8 \times (10 - 45) = -7.5 \times 10^5$ J

(b) $W = 2200 \times 9.8 \times (28 - 10) = 3.9 \times 10^5$ J

(c) $W = 2200 \times 9.8 \times (0 - 28) = -6.0 \times 10^5$ J

Review questions

10. $m = 60$ kg+ 10 kg = 70 kg, $s = 10$ m, $\theta = 0°$

(a) $W = Fs \cos \theta = mgs \cos \theta$

$W = 70 \times 9.8 \times 10 \times \cos 0° = 6\,860 \text{ J} \approx 6.9$ kJ

(b) $P = \frac{W}{t} = \frac{6860}{40} = 171.5\ W \approx 1.7 \times 10^2\ W$

11. (a) $PE_1 = PE_2$ + energy dissipated

$PE_1 = mgh_1 = 2mg$

$PE_2 = mgh_2 = 1.2mg$

$$\text{Percentage dissipated} = \frac{\text{energy dissipated}}{PE_1} \times 100\%$$

$$= \frac{mg\ (2 - 1.2)}{mg\ 2} \times 100\%$$

$$= \frac{0.8}{2} \times 100\% = 40\%$$

(b) The energy is dissipated in the form of heat.

12. (a) $m = 0.1$ kg, $u = 12$ m s^{-1}, $h_i = 0$, $h_f = 5$ m, $d = 5$ m, $v = 0$

$0 = mg(h_f - h_i) + \frac{1}{2}m(v^2 - u^2) + F_f d$

$0 = 0.1 \times 9.8 \times (5 - 0) + \frac{1}{2} \times 0.1 \times (0 - 12^2) + F_f \times 5$

$0 = 4.9 - 7.2 + 5F_f$

$0 = -2.3 + 5F_f$

$2.3 = 5F_f$

$F_f = 0.46$ N

(b) $m = 0.1$ kg, $u = 0$, $h_i = 5$, $h_f = 0$, $F_f = 0.46$, $d = 5$ m

$0 = 0.1 \times 9.8 \times (0 - 5) + \frac{1}{2} \times 0.1 \times (v^2 - 0) + 0.46 \times 5$

$0 = -4.9 + 0.05v^2 + 2.3$

$0 = -2.6 + 0.05v^2$

$2.6 = 0.05v^2$

$v = 7.2$ m s^{-1}

13. (a) $m = 25$ kg, $\Delta h = 1$ m

$E_{gp} = mg\Delta h = 25 \times 9.8 \times 1 = 245$ J

(b) $W = \Delta E_{gp} = 245$ J

(c) If there is no friction, then $W = 245$ J

(d) As the driver would have to work against friction as well as gravity in option B, it would be easier to go with option A, in which gravity alone is being worked against. (However, in terms of mechanical advantage, option B would be better.)

14. (a) $W = \Delta E_{elastic} = F_{elastic}\Delta x = 1500 \times 0.08 = 120$ J

(b) Increase in E_k = release of $E_{elastic}$

$120 = \frac{1}{2}m(v^2 - u^2)$

$120 = \frac{1}{2} \times 30 \times (v^2 - 0)$

$120 = 15v^2$

$v = 2.83$ m s^{-1}

(c) $u = 2.83$ m s^{-1}, $m = 30$ kg, $h_i = 0$, $v = 0$

$0 = mg(h_f - h_i) + \frac{1}{2}m(v^2 - u^2)$

$0 = 30 \times 9.8 \times (h_f - 0) + \frac{1}{2} \times 30 \times (0^2 - 2.83^2)$

$0 = 294h_f - 120$

$120 = 294h_f$

$h_f = 0.41$ m

15. (a) $F_f = 270$ N, $d = 60$ m, $m = 1200$ kg, $v_A = 8$ m s^{-1}, $\Delta h = h_B - h_A = -30$ m

$0 = mg\Delta h + \frac{1}{2}m(v^2 - u^2) + F_f d$

$0 = 1200 \times 9.8 \times -30 + ½ \times 1200 \times (v_B^2 - 8^2) + 270 \times 60$

$0 = -3.53 \times 10^5 + 600v_B^2 - 3.84 \times 10^4 + 1.6 \times 10^4$

$0 = -3.75 \times 10^5 + 600v_B^2$

$3.75 \times 10^5 = 600v_B^2$

$v_B = 25$ m s^{-1}

(b) $W_{friction} = F_f d = 270 \times 60 = 1.6 \times 10^4$ J

$W_{gp} = mg\Delta h = 1200 \times 9.8 \times -30 = 3.53 \times 10^5$ J

Percentage loss = $\frac{1.6 \times 10^4}{3.53 \times 10^5} \times 100\% = 4.5\%$

16. (a) $w_{apparent} = w + ma$

$= 50 \times 9.8 + 50 \times 4$

$= 690$ N

$m_{apparent} = \frac{w_{apparent}}{g} = \frac{690}{9.8} = 70.4$ kg ≈ 70 kg

(b) As there is no change in acceleration, $m_{apparent} = m = 50$ kg

(c) $a = -2$ m s^{-2}

$w_{apparant} = w + ma = (50 \times 9.8) + (50 \times -2) = 390$ N

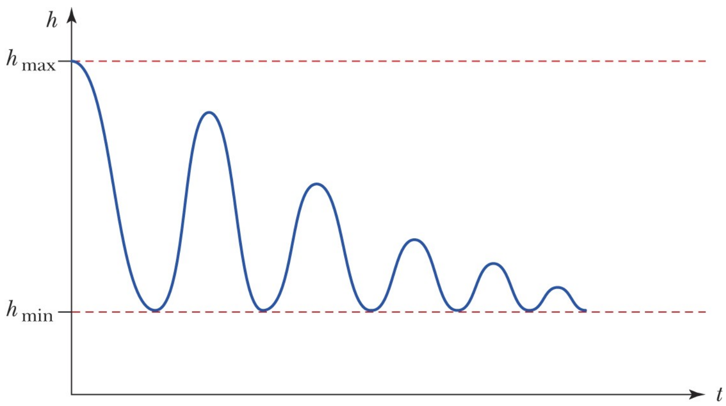
$\mathrm{m}_{apparent} = \frac{w_{apparent}}{g} = \frac{390}{9.8} = 39.8$ kg ≈ 40 kg

(d) $a = -(-2$ m s$^{-2}) = 2$ m s^{-2}

$w_{apparent} = m(g + a) = 50 \times (9.8 + 2) = 590$ N

$m_{apparent} = \frac{w_{apparent}}{g} = \frac{590}{9.8} = 60.2$ kg ≈ 60 kg

17.



19. $m = 1\ 500$ kg, $h_f = 70$ m, $h_i = 0$, $t = 3$ min $= 180$ s, $u = 0$, $v = 4$ m s^{-1}

$$w = mg\,(h_f - h_i) + \frac{1}{2}m(v^2 - u^2)$$

$$= 1\ 500 \times 9.8\ (70 - 0) + \frac{1}{2} \times 1\ 500\ (4^2 - 0)$$

$$= 1.04 \times 10^6 \text{ J}$$

$$P = \frac{W}{t} = \frac{1.04 \times 10^6}{180} = 5.8 \times 10^3 \text{ W} = 5.8 \text{ kW}$$